

MARKSCHEME

November 2011

MATHEMATICAL STUDIES

Standard Level

Paper 2

Deadline for all marking to be completed is 06/12/11 **Ignore deadline on IBIS**

20 pages

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Paper 2 Markscheme Instructions to Examiners

Notes: If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

1 Abbreviations

- **M** Marks awarded for **Method**
- A Marks awarded for an **Answer** or for **Accuracy**
- **R** Marks awarded for clear **Reasoning**
- **G** Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, irrespective of working shown.
- **AG** Answer Given in the question and consequently, marks not awarded.
- ft Marks that can be awarded as **follow through** from previous results in the question.

2 Method of Marking

- (a) All marking must be done in scoris using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.
- (b) If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the A0 annotation, otherwise full annotations must be shown.
- (c) Working crossed out by the candidate should not be awarded any marks.
- (d) Where candidates have written two solutions to a question, only the first solution should be marked.
- (e) If correct working results in a correct answer but then further working is developed, full marks may **not** always be awarded. Full marks **will** be awarded if the candidate shows correct working leading to the correct answer. See also section 4(c).

Example: Calculate the gradient of the line passing through the points (5,3) and (0,9).

Markscheme	Candidates' Scripts	Marking
$ \frac{9-3}{0-5} \qquad (M1) $ Award (M1) for correct substitution in gradient		(MI) (AI)
$= -\frac{6}{5} \tag{A1}$	5 (There is clear understand $y = -\frac{6}{5}x + 9$	
	(ii) $\frac{9-3}{0-5} = -\frac{6}{5}$ $y = -\frac{6}{5}x + 9$	(MI)
		(A0) oout what is required.)

3 Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with '(ft)'.

- (a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.
- (b) If an answer resulting from follow through is extremely unrealistic (e.g. negative distances or incorrect by large order of magnitude) then the final A mark should not be awarded.
- (c) If a question is transformed by an error into a **different**, **much simpler question** then follow through may not apply.
- (d) To award follow through marks for a question part, there must be working present for that part. An isolated follow through answer, without working is regarded as incorrect and receives no marks even if it is approximately correct.
- (e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**
- (f) Inadvertent use of radians will be penalised the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

Markscheme		Candidates' Scripts	Marking
(a) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ (M1)(A1) Award (M1) for substitution in sine rule formula, (A1) for correct substitutions.	(a)	$\frac{\sin A}{4} = \frac{\sin 30}{3}$	(M1)(A0) (use of sine rule but with wrong values)
$A = 22.0^{\circ} (22.0243) (A1)(G2)$		and cannot be a	(A0) I) here was not marked (ft) warded because there was the same question part.)
(b) $x = 7 \tan (22.0243^{\circ})$ (M1) = 2.83 (2.83163) (A1)(ft)	(b)	case (i) $x = 7 \tan 41.8^{\circ}$ = 6.26 case (ii) 6.26	(M1) (A1)(ft) (G0) since no working shown

4 Using the Markscheme

- (a) A marks are **dependent** on the preceding M mark being awarded, it is **not** possible to award (M0)(A1). Once an (M0) has been awarded, all subsequent A marks are lost in that part of the question, even if calculations are performed correctly, until the next M mark. The only exception to this will be for an answer where the accuracy is specified in the question see
 - The only exception to this will be for an answer where the accuracy is specified in the question see section 5.
- (b) A marks are **dependent** on the R mark being awarded, it is **not** possible to award (A1)(R0). Hence the (A1) cannot be awarded for an answer which is correct when no reason or the wrong reason is given.
- (c) In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated G in the mark scheme as an alternative to the full marks. Example (M1)(A1)(A1)(G2).

Example: Using trigonometry to calculate an angle in a triangle.

Markscheme	Candidates' Scripts	Marking
(a) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ (M1)(A1)	$(i) \qquad \frac{\sin A}{3} = \frac{\sin 30}{4}$	(M1)(A1)
Award (M1) for substitution in sine	$A = 22.0^{\circ}$	(A1)
rule formula, (A1) for correct		
substitutions.	(ii) $A = 22.0^{\circ}$	(G2)
	Note: G marks are used only if no v	vorking has been shown
$A = 22.0^{\circ} (22.0243) (A1)(G2)$	and the answer is correct.	

(d) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.

Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' *etc*.

(e) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin \theta}{\cos \theta}$ for $\tan \theta$.

On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.

Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:

the 3 significant figure answer worked through from full calculator display;

the exact value (for example $\sqrt{3}$ if applicable);

the full calculator display in the form 2.83163... as in the example above.

Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

(f) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; 1 · 7; 1,7.

Different descriptions of an interval: 3 < x < 5; (3, 5); (3

Different forms of notation for set properties (e.g. complement): A'; \overline{A} ; A^c ; U - A; $(A; U \setminus A)$.

Different forms of logic notation: $\neg p$; p'; \bar{p} ; \bar{p} ; \bar{p} ; $\sim p$.

 $p \Rightarrow q \; ; \; p \rightarrow q \; ; \; q \Leftarrow p \; .$

(g) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt and exception should be raised through scoris to the team leader.

As from Nov 11 there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

5 Accuracy of Answers

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the candidate's unrounded answer is seen and would round to the required 3 sf answer, then award (A1) and ignore subsequent rounding.

Note: The unrounded answer may appear in either the working box or on the final answer line.

- 2. If the candidate's unrounded answer is **not** seen then award (A1) if the answer given is **correctly** rounded to 2 or more significant figures, otherwise (A0).
- 3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarised in the table below and illustrated in the examples following.

	If candidates final answer is given							
	Exact or correct to 3 or more sf	Incorrect to 3sf	Correct to 2sf ³	Incorrect to 2sf	Correct or incorrect to 1sf			
Unrounded answer seen ¹	Award	Award the final (A1) irrespective of correct or incorrect rounding						
Unrounded answer not seen ²	(A1) (A0) (A1) (A0) (A0)							
Treatment of subsequent parts	As per MS	Treat as follow through, only if working is seen ³						

Examples:

Markscheme			Candidates' Scripts Mark			
9.43 (9.43398)	(A1)	(i)	9.43398 is seen followed by 9; 9.4 9.43; 9.434 etc (correctly rounded)	; <i>(AI)</i>		
		(ii)	9.43398 is seen followed by 9.43399.44 etc (incorrectly rounded)	; <i>(A1)</i>		
		(iii)	9.4	(A1)		
		(iv)	9	(A0) (correct to 1sf)		
		(v)	9.3 (inco	(A0) orrectly rounded to 2sf)		
		(vi)	9.44 (inco	(A0) orrectly rounded to 3sf)		

Markscheme			Candidates' Scripts	Marking
7.44 (7.43798)	(A1)	(i)	7.43798 is seen in the working box followed by 7; 7.4; 7.44; 7.438 etc (correctly rounded)	(A1)
		(ii)	7.43798 is seen followed by 7.437; 7.43 etc (incorrectly rounded)	(AI)
		(iii)	7.4	(A1)
		(iv)	7	(A0) (correct to 1sf)
		(v)	7.5 (incorre	(A0) ectly rounded to 2sf)
		(vi)	7.43 (incorre	(A0) ectly rounded to 3sf)

Example: ABC is a right angled triangle with angle $ABC = 90^{\circ}$, AC = 32 cm and AB = 30 cm. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
(a) BC = $\sqrt{32^2 - 30^2}$ (M1) Award (M1) for correct substitution in	(a) BC = $\sqrt{32^2 - 30^2}$	(M1)
Pythagorus' formula	11 (cm)	(A1)
=11.1 $(\sqrt{124},11.1355)$ (cm) (AI)		ver only seen, but correct)
(b) Area = $\frac{1}{2} \times 30 \times 11.1355$ (M1)	(b) case (i) Area = $\frac{1}{2} \times 30 \times 1$	1 (M1)
Award (M1) for correct substitution in area of triangle formula	2	(working shown)
	$=165 (cm^2)$	(A1)(ft)
= $167(167.032)$ (cm ²) (A1)(ft)		
	case (ii) $= 165 \text{ (cm}^2)$	$(M\theta)(A\theta)(\mathbf{ft})$
	,	g shown, the answer 11 is no marks awarded here)

Rounding of an exact answer to 3 significant figures should be accepted if performed correctly. Exact answers such as $\frac{1}{4}$ can be written as decimals to less than three significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential.

Ratios of π and answers taking the form of square roots of integers or any rational power of an integer (e.g. $\sqrt{13}$, $2^{\frac{2}{3}}$, $\sqrt[4]{5}$,) may be accepted as exact answers. All other powers (e.g. of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In all such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a (\mathbf{ft}) mark regardless of an immediately preceding (M0).

6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final A mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2dp.

Markscheme				Candidates' Scripts	Marking
\$231.62 (231.6189)	(A1)	(i)	231.6		$(A\theta)$
		(ii)	232	(Correct roundin	(A0) ag to incorrect level)
		(iii)	231.61		(A0)
		(iv)	232.00	(Parts ((A0) iii) and (iv) are both
					ling to correct level)

7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final *A* mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for lack of units or incorrect units.

The units are considered only when the numerical answer is awarded (A1) under the accuracy rules given in Section 5.

Example:

Markscheme			Candidates' Scripts			Marking
(a)	$37000\mathrm{m}^2$	(A1)	(a)	$36000\mathrm{m}^2$	(Incorrect answer so u	(A0) nits not considered)
(b)	$3200 \mathrm{m}^3$	(A1)	(b)	$3200 \mathrm{m}^2$		(A0) (Incorrect units)

8 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

QUESTION 1 Units not required in this question

(a) 65 (A1) [1 mark]

(b) (i) $54 (km h^{-1})$ (G2)

Note: If the answer to part (b)(i) is consistent with the answer to part (a) then award (G2)(ft) even if no working seen.

(ii) 19.2 (19.2093...) (G1) [3 marks]

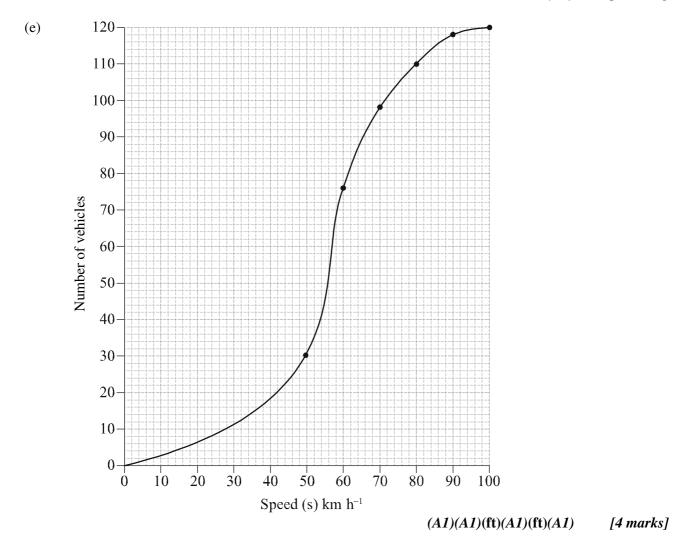
Note: Accept 19, do not accept 20.

(c) 76 (A1) [1 mark]

(d) a = 76, b = 98 (A1)(ft)

Note: Follow through from their answer to part (c) for a and b = their a + 22.

c = 118 (A1) [2 marks]



Question 1 continued

Notes: Award (A1) for axes labelled and correct scales. If the axes are reversed do not award this mark but follow through.

Award (A2)(ft) for their 6 points correct, (A1)(ft) for at least 3 of these points correct.

Award (AI) for smooth curve drawn through all points **including** (0,0). If either the x or the y axis has a break in it to zero, do not award this final mark.

(f) (i) $57 (km h^{-1}) (\pm 2)$

(M1)(A1)(ft)(G2)

Note: Award *(M1)* for clear indication of median on their graph Follow through from their graph.

If their answer is consistent with their incorrect graph but there is no working present on graph then no marks are awarded.

(ii) 90 vehicles (± 2)

(M1)(A1)(ft)(G2)

[4 marks]

Note: Award *(M1)* for clear indication of method on their graph. Follow through from their graph.

If their answer is consistent with their incorrect graph but there is no working present on graph then no marks are awarded.

(g) 50+19.2=69.2

(A1)(ft)

24 (±2) drivers will be fined

(M1)(A1)(ft)(G2)

[3 marks]

Notes: Follow through from their graph and from their part (b)(ii).

Award (M1) for indication of method on their graph.

If their answer is consistent with their incorrect graph but there is no working present on graph then no marks are awarded.

Total [18 marks]

QUESTION 2

(a) (i)
$$\frac{100}{400} \left(\frac{1}{4}, 0.25, 25\% \right)$$
 (A1)

(ii)
$$\frac{90}{400} \left(\frac{9}{40}, 0.225, 22.5 \% \right)$$
 (A1)

(iii)
$$\frac{20}{400} \left(\frac{1}{20}, 0.05, 5\% \right)$$
 (A1)(A1)

Note: Award (A1) for numerator, (A1) for denominator.

(iv)
$$\frac{120}{400} \left(\frac{3}{10}, 0.3, 30 \% \right)$$
 (A1)(A1)

Note: Award (A1) for numerator, (A1) for denominator.

(v)
$$\frac{30}{110} \left(\frac{3}{11}, 0.273, 27.3 \% \right)$$
 (0.272727...) (A1)(A1) [8 marks]

Note: Award (A1) for numerator, (A1) for denominator. Accept 0.27, do not accept 0.272, do not accept 0.3.

(b)
$$\frac{1}{20} \neq \frac{1}{4} \times \frac{9}{40}$$
 (R1)(ft)

Note: The fractions must be used as part of the reason. Follow through from (a)(i), (a)(ii) and (a)(iii).

Pam is not correct (A1)(ft) [2 marks]

Notes: Do not award $(R\theta)(A1)$. Accept the events are not independent (dependent).

Question 2 continued

(c) (i) The mathematics course and language of examination are independent.

(A1)

Notes: Accept "There is no association between Mathematics course and language".

Do not accept "not related", "not correlated", "not influenced".

(ii)
$$\frac{110}{400} \times \frac{150}{400} \times 400 \left(= \frac{110 \times 150}{400} \right)$$
 (M1)

$$=41.25$$
 (A1) $=41.3$ (AG)

Note: 41.25 and 41.3 must be seen to award final (A1).

(d) (i) 7.67 (7.67003...) (G2)

Note: Accept 7.7, do not accept 8 or 7.6.

Award (G1) if formula with all nine terms seen but their answer is not one of those above.

(ii) 4 (G1)

(iii) 9.488 (A1)(ft) [4 marks]

Notes: Accept 9.49 or 9.5, do not accept 9.4 or 9. Follow through from their degrees of freedom.

(e) 7.67 < 9.488

OR

$$p = 0.104..., p > 0.05$$
 (R1)

Accept (Do not reject) H_0 (Pam's belief is correct) (A1)(ft) [2 marks]

Notes: Follow through from part (d). Do not award (R0)(A1).

Total [19 marks]

[3 marks]

QUESTION 3

(a)
$$\cos ADB = \frac{12^2 + 20^2 - 28^2}{2(12)(20)}$$
 (M1)(A1)

Notes: Award (M1) for substituted cosine rule formula, (A1) for correct substitutions.

$$\angle ADB = 120^{\circ}$$
 (A1)(G2) [3 marks]

(b) Area =
$$\frac{(12)(20)\sin 120^{\circ}}{2}$$
 (M1)(A1)(ft)

Notes: Award (M1) for substituted area formula, (A1)(ft) for their correct substitutions.

=
$$104 \text{ cm}^2$$
 ($103.923...\text{cm}^2$) (A1)(ft)(G2) [3 marks]

Note: The final answer is 104 cm², **the units are required**. Accept 100 cm².

(c)
$$\frac{\sin BCD}{12} = \frac{\sin 60^{\circ}}{13}$$
 (A1)(ft)(M1)(A1)

Note: Award (A1)(ft) for their 60 seen, (M1) for substituted sine rule formula, (A1) for correct substitutions.

$$BCD = 53.1^{\circ}$$
 (53.0736...) (A1)(G3) [4 marks]

Note: Accept 53, do not accept 50 or 53.0.

Question 3 continued

(d) Using triangle ABC

$$\frac{\sin BAC}{13} = \frac{\sin 53.1^{\circ}}{28}$$

 $(M1)(A1)({\rm ft})$

OR

Using triangle ABD

$$\frac{\sin BAD}{12} = \frac{\sin 120^{\circ}}{28}$$

(M1)(A1)(ft)

Note: Award (MI) for substituted sine rule formula (one of the above), $(AI)(\mathbf{ft})$ for their correct substitutions. Follow through from (a) or (c) as appropriate.

$$BAC = BAD = 21.8^{\circ}$$
 (21.7867...)

(A1)(ft)(G2)

Notes: Accept 22, do not accept 20 or 21.7.

Accept equivalent methods, for example cosine rule.

$$180^{\circ} - (53.1^{\circ} + 21.8^{\circ}) \neq 90^{\circ}$$
, hence triangle ABC is not right angled

(R1)(AG)

OR

$$\frac{\text{CD}}{\sin 66.9^{\circ}} = \frac{13}{\sin 60^{\circ}}$$

 $(M1)(A1)(\mathrm{ft})$

Note: Award (M1) for substituted sine rule formula, (A1)(ft) for their correct substitutions. Follow through from (a) and (c).

$$CD = 13.8 \quad (13.8075...)$$

(A1)(ft)

$$13^3 + 28^2 \neq 33.8^2$$
, hence triangle ABC is not right angled.

(R1)(ft)(AG) [4 marks]

Note: The complete statement is required for the final (*R1*) to be awarded.

Total [14 marks]

QUESTION 4

(a) (i) x = 0 (A1)(A1)

Note: Award (A1) for x = a constant, (A1) for the constant in their equation being 0.

(ii)
$$-1.58 \ (-1.58454...)$$
 (G1)

-17-

Note: Accept -1.6, do not accept -2 or -1.59.

Note: Award at most (G1)(G0) if brackets not used. Award (G0)(G1)(ft) if coordinates are reversed.

Note: Accept x = 2.06, y = 4.49.

Note: Accept 2.1, do not accept 2.0 or 2. Accept 4.5, do not accept 5 or 4.50.

(b)
$$f'(x) = 2x - 2 - \frac{9}{x^2}$$
 (A1)(A1)(A1) [4 marks]

Notes: Award (A1) for 2x, (A1) for -2, (A1) for -9, (A1) for x^{-2} . Award a maximum of (A1)(A1)(A1)(A0) if there are extra terms present.

(c)
$$f'(x) = \frac{x^2(2x-2)}{x^2} - \frac{9}{x^2}$$
 (M1)

Note: Award (M1) for taking the correct common denominator.

$$=\frac{(2x^3-2x^2)}{x^2}-\frac{9}{x^2} \tag{M1}$$

Note: Award (M1) for multiplying brackets or equivalent.

$$=\frac{2x^3 - 2x^2 - 9}{x^2}$$
 (AG) [2 marks]

Note: The final (M1) is not awarded if the given answer is not seen.

(d)
$$f'(1) = \frac{2(1)^3 - 2(1) - 9}{(1)^2}$$
 (M1)
= -9 (A1)(G2) [2 marks]

Note: Award (M1) for substitution into given (or their correct) f'(x). There is no follow through for use of their incorrect derivative.

Question 4 continued

(e)
$$\frac{1}{9}$$
 (A1)(ft) [1 mark]

Note: Follow through from part (d).

(f)
$$y-8=\frac{1}{9}(x-1)$$
 (M1)(M1)

Notes: Award (M1) for substitution of their gradient from (e), (M1) for substitution of given point. Accept all forms of straight line.

$$y = \frac{1}{9}x + \frac{71}{9}$$
 (y = 0.1111111...x + 7.88888...) (A1)(ft)(G3) [3 marks]

Note: Award the final (A1)(ft) for a correctly rearranged formula of their straight line in (f).

Accept 0.11x, do not accept 0.1x.

Accept 7.9, do not accept 7.88. do not accept 7.8.

Notes: Follow through from their line L from part (f) even if no working shown

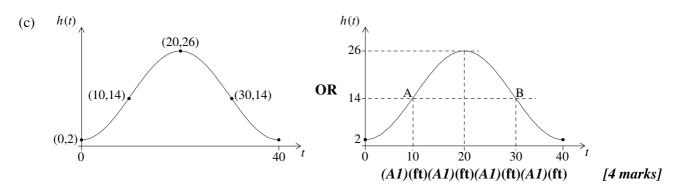
Award at most (A0)(A1) (ft) if their correct coordinate pairs given.

Note: Accept –2.5, do not accept –2.49. Accept 3.6, do not accept 3.60.

Total [19 marks]

QUESTION 5 Units not required in this question

- (a) (i) 14 m (A1)
 - (ii) 26 m (A1) [2 marks]
- (b) A:10, B:30 (A1)(A1) [2 marks]



Note: Award *(A1)*(**ft**) for coordinates of each point clearly indicated either by scale or by coordinate pairs. Points need not be labelled A and B in the second diagram.

Award a maximum of (A1)(A0)(A1)(ft)(A1)(ft) if coordinates are reversed. Do not penalise reversed coordinates if this has already been penalised in Q4(a)(iii).

(d) (i)
$$a = -12$$
 (A2)(ft)

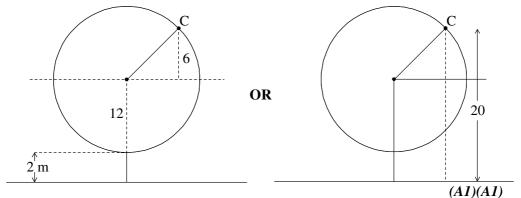
Note: Follow through from (a) even if no working seen.

(ii)
$$b = \frac{360}{40}$$
 (M1)
= 9 (A1)(G2)

(iii)
$$c = 14$$
 (A1) [5 marks]

Question 5 continued

(e) (i)



Note: Award (A1) for C in correct quadrant, (A1) for clear indication of 20 m above ground.

(ii)
$$90 + \sin^{-1}\left(\frac{6}{12}\right)$$
 (M1)(M1)

Note: Award (*M1*) for adding 90, (*M1*) for use of trigonometric ratio with appropriate values.

$$=120^{\circ} \tag{A1)(G2)}$$

OR

$$20 = -12\cos(9T) + 14 \tag{M1}(M1)$$

Note: Award (M1) for substituting values, (M1) for equating to 20.

$$9T = 120^{\circ} \tag{A1)ft(G2)}$$

Note: Follow through from their a, b and c in part (d). The final answer must be an obtuse angle. 9T does not have to be seen.

(iii)
$$\frac{120}{360} \times 40$$
 (M1)
= $13\frac{1}{3}$ seconds (13.3333...) (A1)(ft)(G2)

OR

$$9T = 120^{\circ}$$
 (M1)
 $T = 13\frac{1}{3}$ (A1)(ft)(G2) [7 marks]

Notes: Follow through from their answer to part (e)(ii). The final answer must be consistent with their diagram. Accept 13.

Total [20 marks]